Distortion Level Analysis of a 2D Median Filter with a Weighted Central Element

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Аннотация. **В статье показана оценка применения медианных фильтров с взвешенными центральными элементами для очистки изображений от импульсного шума. Показано влияние вероятности получения неискаженных пикселей и искажения функции распределения пикселей изображения на результат обработки. Продемонстрировано, что медианные фильтры с окном 3×3 позволяют получать более высокое качество очистки изображений от шума с небольшой вероятностью импульсного шума, а медианные фильтры с размерами окон 5×5 и 7×7 позволяют получать более высокое качество очистки изображений от шума с высокой вероятностью импульсного шума. Полученный результат может быть использован в обработке медицинских изображений и адаптивных системах фильтрации при обработке фото- и видеоданных.**

Ключевые слова: цифровая обработка изображений, импульсный шум, медианный фильтр, взвешенный центральный элемент, пиковое отношение сигнал/шум

I. Introduction

In the process of transmission and conversion through radio systems, images are exposed to various interference, which in some cases leads to deterioration of visual quality and loss of image areas. With the widespread introduction of digital communication systems increases the urgency of solving the problem of restoring images obtained with the help of photo and video cameras, in order to filter images. In practice, often there are images distorted by noise, which appears at the stages of its formation and transmission over the communication channel. Linear filtration algorithms are applied for the restoration and improvement of the visual quality of the images. It can be used to reduce noise in images. However, in order to suppress the noise and at the same time keep the contour part of the images, it is necessary to use nonlinear filters.

All linear filtering algorithms lead to the smoothing of sharp brightness drops of processed images. The disadvantage is that linear procedures are optimal for Gaussian distribution of signals, noise, and observed data. Real images do not obey this probability distribution. One of the main reasons for this is that images have a variety of borders, brightness differences, transitions from one texture to another. Succumbing to the local Gauss to a description within limited areas, a real image in this respect, poorly presented as globally Gaussian objects. This is the reason for the poor transmission of boundaries in linear filtering. The second feature of linear filtering is its optimality, as just mentioned, in the case of Gaussian interference. Usually, this condition is met by noise in the images, so when they are suppressed, linear algorithms have high performance.

However, you often have to deal with images that are distorted by other types of interference. One of them is impulse noise. Under its influence on the image observed white and black dots randomly scattered around the frame. The use of linear filtering, in this case, is inefficient – since each of the input pulses gives a response in the form of a pulse filter characteristics, and their combination contributes to the propagation of noise over the entire area of the frame.

A successful solution to these problems is the use of median filtering, developed in [1]. Median filtering is used in infrared image processing [2], images were taken underwater [3], and also in medicine when cleaning x-rays from noise [4], images obtained by magnetic resonance imaging [5] and images the retina of the eye [6]. In addition, median filters (MF) are used in adaptive image filtering systems [7].

The median filter can be designed to remove noise in the best way, but a MF with a weighted center element (WCE) gives similar results, and by selecting the weight of the center element of the filter the user can get the desired level of smoothing [8].

Because the center weight is reduced to achieve the desired level of impulse suppression, the output image will experience increased distortion, especially around small parts of the image. However, the application of MV with WCE can be very effective in eliminating the “salt and pepper” noise while keeping the image details [9].

In this paper, we show the dependence of the image reconstruction quality distorted by pulse noise on its intensity and the weight of the MF with the WCE.

II. Cleaning the Image Impulse Noise

Impulse noise refers to the distortion of the signal by pulses, that is, emissions with very large positive or negative values and short duration. When processing images, pulse noise occurs, for example, due to decoding errors that lead to black and white dots on the image. Therefore, it is often called point noise. Noise emissions are particularly noticeable in very dark or very light areas of images. For such areas, you can display formulas for the probability of correct reproduction [10].

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The appearance of noise emission at each point (i,j) of the image has a probability p and does not depend on the presence of noise at other points of the image or on the original image. The distorted point acquires a stable value d, corresponding to black or white.

Let $\{X(i,j)\}$ is a distorted image.

Then

$$X(i,j) = \begin{cases} S(i,j) \text{ with probability } (1-p), \\ d \text{ with probability } p, \end{cases} \quad (1)$$

where $\{S(i,j)\}$ – the values of the undistorted image.

Let N equal to the maximum possible pixel value of the image. For example, for 8-bit grayscale images $N = 255$. Assuming that the appearance of white or black pixels is equally probable in pulsed noise, the pulsed noise model can be rewritten as:

$$Pr\{X(i,j) = 0\} = \frac{p}{2}, Pr\{X(i,j) = N\} = \frac{p}{2}, \\ Pr\{X(i,j) = S(i,j)\} = 1 - p. \quad (2)$$

where $Pr\{A\}$– the probability of the event A.

When considering a median filter with window size $2L+1$ and the weight of the central element $2K+1$ when calculating the result of the filter operation, the median is searched by the array consisting of $2L+2K+1$ elements. If the central pixel of the part of the image to which the MF with the WCE is applied has the minimum possible value 0, then it is necessary and sufficient that the filter result is 0 among the remaining ones $2L$ pixels of the image at least $(2L-2K-1)/2$ pixels had values 0. Since the number of pixels is an integer, this requirement can be replaced by the requirement of equal to 0 at least $L-K$ pixels of the image. If the Central pixel of the part of the image to which the MF with the WCE is applied has a value other than 0, then in order for the filter to work, it is necessary and sufficient that among $2L$ the remaining pixels, the parts of the image $(2L+2K+1)/2$ have at least 0 pixels. Since the number of pixels is an integer, this requirement can be replaced by a requirement of at least 0 $L+K+1$ pixels of the image part.

Thus, the probability of obtaining 0 as a result of the work of the MF with the WCE is equal

$$PR\{Y(i,j) = 0\} = \frac{p}{2}\sum_{i=L-K}^{2L} C_{2L}^{i}\left(\frac{p}{2}\right)^{i}\left(1-\frac{p}{2}\right)^{2L-i} + \\ +\left(1-\frac{p}{2}\right)\sum_{i=L+K+1}^{2L} C_{2L}^{i}\left(\frac{p}{2}\right)^{i}\left(1-\frac{p}{2}\right)^{2L-i}. \quad (3)$$

Taking into account the assumption of equal probability of appearance of black and white pixels at pulse noise, it is possible to write a similar formula for the probability of obtaining as a result of the work of MF with WCE:

$$PR\{Y(i,j) = N\} = \frac{p}{2}\sum_{i=L-K}^{2L} C_{2L}^{i}\left(\frac{p}{2}\right)^{i}\left(1-\frac{p}{2}\right)^{2L-i} + \\ +\left(1-\frac{p}{2}\right)\sum_{i=L+K+1}^{2L} C_{2L}^{i}\left(\frac{p}{2}\right)^{i}\left(1-\frac{p}{2}\right)^{2L-i}. \quad (4)$$

Taking into account the formulas (3) and (4), we can find the probability of obtaining a “distorted” result of the MF with the WCE corresponding to obtaining 0 or at the output of the filter and the “undistorted” result of the MF with the WCE corresponding to obtaining values other than 0 or at the output of the filter.

$$Pr\{Y(i,j) - \text{"}distorted\text{"}\} = \\ = p\sum_{i=L-K}^{2L} C_{2L}^{i}\left(\frac{p}{2}\right)^{i}\left(1-\frac{p}{2}\right)^{2L-i} + \\ +(2-p)\sum_{i=L+K+1}^{2L} C_{2L}^{i}\left(\frac{p}{2}\right)^{i}\left(1-\frac{p}{2}\right)^{2L-i}, \quad (5)$$

$$Pr\{Y(i,j) - \text{"}undistorted\text{"}\} = \\ = 1 - p\sum_{i=L-K}^{2L} C_{2L}^{i}\left(\frac{p}{2}\right)^{i}\left(1-\frac{p}{2}\right)^{2L-i} - \\ -(2-p)\sum_{i=L+K+1}^{2L} C_{2L}^{i}\left(\frac{p}{2}\right)^{i}\left(1-\frac{p}{2}\right)^{2L-i}. \quad (6)$$

III. ASSESSMENT OF SUPPRESSION OF IMPULSE NOISE

To estimate the structural change of the image after the application of MF with the WCE assume that the pixels of the image are subject to a uniform distribution law with the distribution function

$$F_X(x) = \begin{cases} 0, \text{if } x < 0, \\ \frac{x}{N}, \text{if } 0 \le x < N, \\ 1, \text{if } x \ge N. \end{cases} \quad (7)$$

where x is the brightness level of the pixel of the image.

Then the function of distribution of pixels of the image distorted by pulse noise will have the form

$$F_X(x) = \begin{cases} 0, \ if\ x < 0, \\ \frac{p}{2} + \frac{(1-p)x}{N}, \quad if\ 0 \le x < N, \\ 1, if\ x \ge N. \end{cases} \quad (8)$$

To assess the degree of difference in function $F_Y(x)$ and $F_X(x)$ we will use the values I and T:

$$I = \int_0^N |F_Y(x) - F_X(x)|dx. \quad (9)$$

$$T = \int_0^N \left|F(x) - F_y(x)\right| dx \quad (10)$$

where $F(x)$ the distribution function of pixels to blur the image. The graph of the function (7) on the interval $(0, N)$ for $p = 0{,}1$ and $N = 255$ (the case of 8-bit grayscale images) shown in Fig. 1 (a). The result of the application of MF with the WCE distorts the processed image. To estimate the level of this distortion, it is necessary to investigate function $F_Y(x)$ from formula (8) taking into account the substitution of the function $F_X(x)$ from formula (9). In Fig. 1(b)–1(f) the graph of functions is shown $F_Y(x)$, obtained for the case $p = 0{,}1$, $N = 255$, $L = 4$ (the median filter 3 × 3) with different values $K = 0; 1; 2; 3; 4$.

From Fig. 1(b)–1(f) it can be seen that for small values the distribution K function changes $F_Y(x)$ significantly with respect to the function $F_X(x)$, and with increasing the K difference between these functions becomes smaller and smaller.

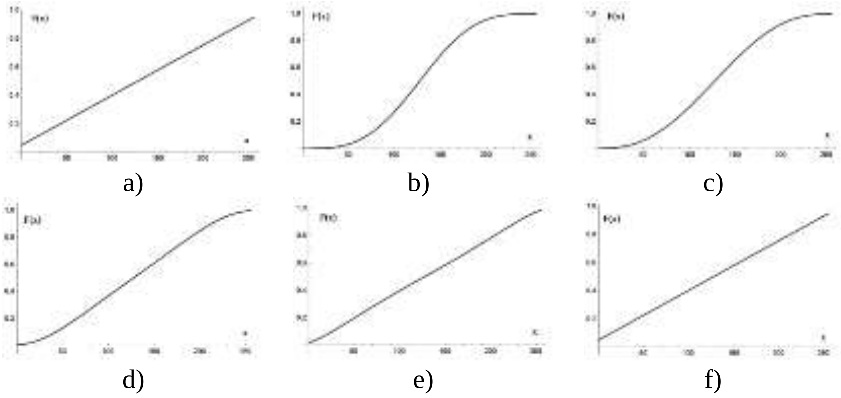

Fig. 1. Graphs of distribution functions for grayscale images $N = 255$ with MF $L = 4$ and treated with different weighting factors of the central element: a) noisy image $p = 0.1$; b) $K = 0$; c) $K = 1$; d) $K = 2$; e) $K = 3$; f) $K = 4$

The geometric meaning of this integral I is shown in Fig. 2. It is equal to the area of the figure enclosed between the graphs of the function $F_Y(x)$ and $F_X(x)$. The smaller the value I the closer the functions are $F_Y(x)$ and $F_X(x)$ to each other in terms of the metric of the functional space [9].

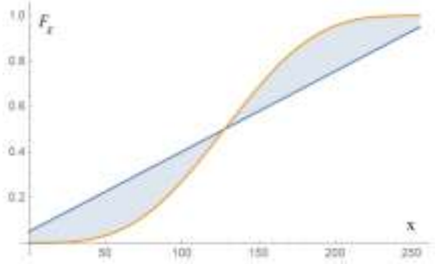

Fig. 2. The geometric meaning of the value I for the case $N = 255$, $p = 0.1$, $L = 4$, $K = 0$

Simulation of the calculations of the formulas (9), (10) over the noisy and restored images for some images. To estimate the noise suppression by the formula (14) it is necessary to compare the cumulative histogram of some image noisy with pulse noise with different intensity with the cumulative histogram of the same restored image. To simulate a cumulative histogram, it is necessary to simulate histograms of noisy and restored images.

The results of the calculations of formula (9) for the median filter with a 7×7 window and the weight of the central element $2K + 1$ are given in Table I. The results of the calculations of formula (10) for the median filter with a 7×7 window and the weight of the central element $2K + +1$ are given in Table II.

TABLE I. VALUES I FOR THE MEDIAN FILTER WITH A WINDOW 7×7 AND THE WEIGHT OF THE CENTRAL ELEMENT $2K + 1$

p	K						
	1	3	5	7	9	11	13
0,01	1.0e+09 *	5.1124	5.0002	4.8726	4.7272	4.5724	4.4228
0,1	1.0e+10 *	2.5593	2.5251	2.4815	2.4323	2.3800	2.3213
0,25	1.0e+11 *	1.4053	1.3966	1.3851	1.3720	1.3567	1.3404
0,5	1.0e+11 *	5.5326	5.5167	5.4775	5.4127	5.3051	5.1370
0,75	1.0e+11 *	9.8965	9.5337	8.5488	7.0915	5.4122	3.7198
p	K						
	15	17	19	21	23	25	27
0,01	4.2709	4.1135	3.9543	3.8080	3.6757	3.5421	3.4324
0,1	2.2612	2.1966	2.1324	2.0664	1.9345	1.8674	1.8032
0,25	1.3224	1.3033	1.2826	1.2594	1.2325	1.2000	1.1549
0,5	4.8707	4.4706	3.9028	3.1765	2.3585	1.5561	0.8881
0,75	2.2859	1.2370	0.5866	0.2421	0.0905	0.0377	0.0252

p	K						
	29	31	33	35	37	39	41
0,01	3.3381	3.2749	3.2263	3.1932	3.1753	3.1753	3.1901
0,1	1.7389	1.6740	1.6043	1.5216	1.4017	1.1877	0.8541
0,25	1.0843	0.9774	0.8144	0.5996	0.3700	0.1778	0.0664
0,5	0.4289	0.1730	0.0635	0.0304	0.0258	0.0277	0.0290
0,75	0.0241	0.0253	0.0261	0.0262	0.0262	0.0262	0.0262

p	K		
	43	45	47
0,01	3.2191	3.2610	3.3229
0,1	0.4860	0.2867	0.3296
0,25	0.0289	0.0274	0.0333
0,5	0.0295	0.0297	0.0297
0,75	0.0262	0.0262	0.0262

TABLE II. VALUES T FOR THE MEDIAN FILTER WITH A WINDOW 7×7 AND THE WEIGHT OF THE CENTRAL ELEMENT $2K + 1$

p	K						
	1	***3***	***5***	***7***	***9***	***11***	***13***
0,01	1.0e+11 *	7.8019	7.8096	7.8170	7.8243	7.8342	7.8455
0,1	1.0e+11 *	7.8003	7.8073	7.8160	7.8244	7.8349	7.8467
0,25	1.0e+12 *	0.7802	0.7810	0.7818	0.7830	0.7843	0.7857
0,5	1.0e+12 *	0.7833	0.7844	0.7869	0.7907	0.7971	0.8082
0,75	1.0e+11 *	9.8965	9.5337	8.5488	7.0915	5.4122	3.7198
p	K						
	15	17	19	21	23	25	27
0,01	7.8569	7.8696	7.8809	7.8931	7.9065	7.9184	7.9315
0,1	7.8591	7.8718	7.8852	7.8999	7.9158	7.9325	7.9493
0,25	0.7874	0.7891	0.7911	0.7933	0.7965	0.8003	0.8064
0,5	0.8269	0.8574	0.9061	0.9805	1.0893	1.2345	1.4082
0,75	2.2859	1.2370	0.5866	0.2421	0.0905	0.0377	0.0252
p	K						
	29	31	33	35	37	39	41
0,01	7.9447	7.9588	7.9725	7.9860	7.9992	8.0126	8.0258
0,1	7.9668	7.9850	8.0047	8.0282	8.0615	8.1205	8.2291
0,25	0.8173	0.8344	0.8641	0.9128	0.9817	1.0670	1.1579
0,5	1.5940	1.7644	1.9053	2.0021	2.0574	2.0877	2.0996
0,75	0.0241	0.0253	0.0261	0.0262	0.0262	0.0262	0.0262

p	K		
	43	45	47
0,01	8.0389	8.0519	8.0671
0,1	8.4448	8.8065	9.2439
0,25	1.2346	1.2863	1.3110
0,5	2.1035	2.1043	2.1045
0,75	0.0262	0.0262	0.0262

TABLE III. VALUES PSNR FOR THE MEDIAN FILTER WITH A WINDOW AND THE WEIGHT OF THE CENTRAL ELEMENT $2K + 1$

Window size	Noise intensity levels, p				
	0,01	0,1	0,25	0,5	0,75
3×3	10.1504	9.3896	8.3382	7.0111	6.0128
5×5	10.1378	9.3798	8.3317	7.0070	6.0112
7×7	10.2471	9.3708	8.3233	7.0036	6.0099

The values given in Table I show a decrease or no change in the values in each row, which indicates an improvement in the image recovery quality or no change in the improvement from any step. The values given in Table II show an increase or no change in the values in each row, which indicates an improvement in the image recovery quality or no change in the improvement from any step. The values PSNR for the median filter with a window and the weight of the central element 2K+1 given in Table III.

The results obtained during the simulation are presented in the form of graphs in Fig. 3–4. A clearly visible increase in the quality of cleaning noise reaching the maximum value and the subsequent reduction in the quality of processing, with a change K from 0 to L, in all cases. This is due to the fact that for small noise levels, the preservation of the image structure plays an important role, the changes of which depend on the value I given in Table I. The decrease in processing quality for large values K is due to a significant decrease in the probability of obtaining an undistorted image pixel. For high noise levels, the increase in processing quality to the maximum value I by reducing the value is not so noticeable due to a significant increase in the probability of obtaining an undistorted image pixel.

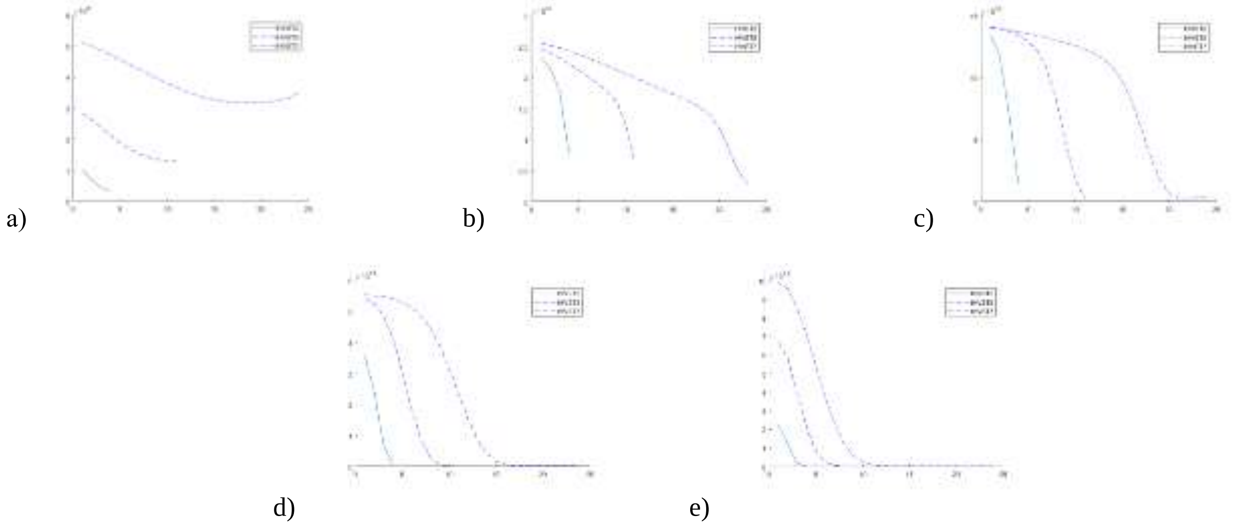

Fig. 3. The results of the MF processing image with different levels of impulse noise: a) $p = 0.01$; b) $p = 0.1$; c) $p = 0.25$; d) $p = 0.5$; e) $p = 0.75$.

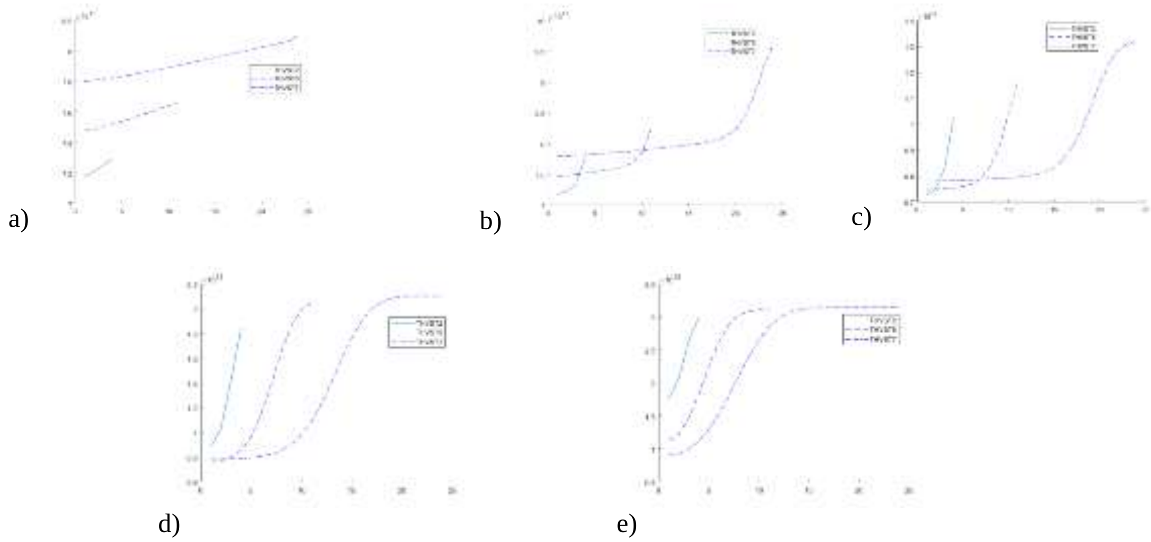

Fig. 4. The results of the MF processing image with different levels of impulse noise: a)$p = 0.01$; b) $p = 0.1$; c) $p = 0.25$; d) $p = 0.5$; e) $p = 0.75$.

Fig. 4 clearly shows the decrease in the quality of the initial values from the noise to the maximum value and the subsequent decrease in the quality of processing, when changing K from 0 to L, in all cases. This is due to the fact that for small noise levels, the preservation of the image structure plays an important role, the changes of which depend on the value T given in Table II.

Tables I and II show the values I and T of both the formulas (9) and (10) for grayscale $N = 255$ images with impulse noise with probabilities $p = 0.01; 0.05; 0.1; 0.25; 0.5; 0.75$, and median filters with a mask 7×7 with different weights of the central coefficient. Table II shows that the increase in the weight of the central element leads K to a decrease in the value I and an increase T in the value in all cases. This means that the use of large coefficients of the central element in the MF with the WCE less distorts the structure of the image, compared with small values.

IV. Conclusion

We show the evaluation of the use of MF with WCE for cleaning images from impulse noise. The influence of the probability of obtaining undistorted pixels and distortion of the image pixel distribution function on the processing is shown. It is demonstrated that MF with a window 3×3 allow obtaining a higher quality of image cleaning from noise with a small probability of impulse noise, and MF with window sizes 5×5 and 7×7 allow to obtain a higher quality of image cleaning from noise with a high probability of impulse noise. The obtained result can be used in medical image processing and adaptive filtering systems for photo and video data processing.

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